Three experiments used in an Introductory Class in Electromagnetics and EMC for Junior-Level Computer Engineers

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Experiment #1

Use of Common-Mode Choke in DC-DC Converter Design
Goals of this experiment:

• Measure self-inductance using series-resonance method and compare with predicted value.
• Understand operation of common-mode choke.
• Measure the self-inductance $L$ and mutual inductance $M$ of a common-mode choke.
• Analyze and construct a simple dc-dc switching converter. (This goal ties this EMC course to the electronics courses which are prerequisite for this class.)
• Measure its conversion efficiency at different switching rates.
• Verify common-mode choke reduces common-mode currents on power cable of dc-dc converter.
• Observe how common-mode choke reduces radiated emissions on ac power cord of dc-dc converter.
Overview

• A homemade common-mode choke is characterized in terms of L and M.
• A simple switching DC-DC converter is built from discrete components, and its operation is analyzed.
• Its conversion efficiency is measured at different switching frequencies.
• Common-mode currents flowing on the dc power cable are measured using a current probe both with and without the common-mode choke.
• Also, conducted emissions on the 120 VAC power line are measured with a “Line Impedance Stabilization Network” (LISN) both with and without the common-mode choke.
Clear benefits of using the common-mode choke will be demonstrated using

1. Current probe to measure common-mode currents on the dc power cable

2. LISN to measure conducted emissions on the ac power cable.
Lab 1 Equipment List

- Agilent E4402B ESA-E Series 100 Hz – 3 GHz Spectrum Analyzer
- EMCO Model 3810/2 LISN (9 kHz – 30 MHz)
- Agilent 54624A 100 MHz Digital Oscilloscope (with 2 scope probes)
- EG&G SCP-5(I)HF (125 kHz – 500 MHz) Snap On Current Probe
- Agilent E3631A Triple Output DC Power Supply (5 V at 5 A)
- Agilent 33250A 80 MHz Function Generator
Common-Mode Choke Construction and Measurements
Measuring L and M for a “Home-made” Common-Mode Choke

- Common-mode choke constructed by bifilar winding 20 turns of 2 strands of 20-gage hookup wire around a toroidal core.
- Toroidal Core has:
  - Outer diameter = 2.5 cm
  - Inner diameter = 1.0 cm
  - Thickness = 0.9 cm
  - relative permeability $\mu_R = 5000$. 

![Diagram of common-mode choke with bifilar winding and toroidal core dimensions annotated.](image)
Self Inductance $L$ of either toroidal coil may be approximately calculated using:

$$L_{\text{toroid}} = \mu_0 \cdot \mu_R \cdot N^2 \cdot \frac{\text{thickness}}{2 \cdot \pi} \cdot \ln \left( \frac{\text{outer\_diameter}}{\text{inner\_diameter}} \right)$$
Approximate Calculation of Self-Inductance “L” of either coil in choke

Where:
\[ \mu_0 := 4\cdot\pi\cdot10^{-7} \cdot \frac{H}{m} \]
For the toroidal core used:
\[ \mu_R := 5000 \]

\[ N := 20 \text{ turns} \]
\[ \text{thickness} := 0.9\cdot10^{-2} \cdot m \]
\[ \text{outer\_diameter} := 2.5\cdot10^{-2} \cdot m \]
\[ \text{inner\_diameter} := 1.0\cdot10^{-2} \cdot m \]

\[ L_{\text{toroid}} := \mu_0 \cdot \mu_R \cdot N^2 \cdot \frac{\text{thickness}}{2 \cdot \pi} \cdot \ln\left( \frac{\text{outer\_diameter}}{\text{inner\_diameter}} \right) \]

\[ L_{\text{toroid}} = 3.299 \times 10^{-3} H \]
Experimental Measurement of Self-Inductance “L”

- Use the *series LC resonance* method... Open-circuit one coil, while other coil is resonated with a known value of capacitance.

![Diagram](image)

Function Generator 0-80MHz

The frequency “$f_{null}$” where the signal null occurs is the frequency at which

\[
\text{Inductor Impedance} + \text{Capacitor Impedance} = 0
\]

\[
j \cdot 2 \cdot \pi \cdot f_{null} \cdot L_{\text{unknown}} + \frac{1}{j \cdot 2 \cdot \pi \cdot f_{null} \cdot C_{\text{known}}} = 0
\]

\[
f_{null} = \frac{1}{2 \cdot \pi \cdot \sqrt{L_{\text{unknown}} \cdot C_{\text{known}}}}
\]

\[
L_{\text{unknown}} = \frac{1}{\left(4 \cdot C_{\text{known}} \cdot f_{null}^2 \cdot \pi^2\right)}
\]

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As expected, both coils were found to have the same $f_{\text{null}}$ value, and hence both coils had the same self inductance.

An LCR meter (Extech Model 380193) was also used to measure the self-inductance of each coil in the common-mode choke, and each coil read 3.51 mH on the LCR meter.
Measuring the Mutual Inductance “M”

Leq(AC) = 2(L - M)  \hspace{1cm} \text{(Core flux set up by each coil cancels)}

Leq(AD) = 2(L + M)  \hspace{1cm} \text{(Core flux set up by each coil reinforces)}
Finding $\text{Leq}(AB)$

In this case

\[ C_{\text{known}} := 10 \cdot 10^3 \cdot \text{pF} \quad f_{\text{null}} := 2.071 \cdot \text{MHz} \]

\[ \text{Leq}_{AB} := \frac{1}{\left(4 \cdot C_{\text{known}} \cdot f_{\text{null}}^2 \cdot \pi^2 \right)} \quad \text{Leq}_{AB} = 5.9058181 \times 10^{-7} \text{H} \]

Finding $\text{Leq}(CD)$

In this case

\[ C_{\text{known}} := 10 \cdot 10^3 \cdot \text{pF} \quad f_{\text{null}} := 13.0 \cdot \text{kHz} \]

\[ \text{Leq}_{CD} := \frac{1}{\left(4 \cdot C_{\text{known}} \cdot f_{\text{null}}^2 \cdot \pi^2 \right)} \quad \text{Leq}_{CD} = 0.0149883 \text{H} \]
Solving for \( L \) and \( M \)

**Given**

\[
5.906 \times 10^{-7} = 2 \cdot (L - M)
\]

\[
0.015 = 2 \cdot (L + M)
\]

**Find** \((L, M) = \left(\begin{array}{c}
3.7501477 \times 10^{-3} \\
3.7498523 \times 10^{-3}
\end{array}\right)\)
Conclusions

• The closer M is to L, the better the common-mode choke. Here \( L = 3.7502 \text{ mH} \) and \( M = 3.7499 \text{ mH} \), so they are very close!

• This common-mode choke exhibits a very low equivalent inductance of \( \text{Leq} = 2(L-M) = 600 \text{ nH} \) to differential mode currents (which are usually the desired signal).

• It exhibits a much higher inductance of \( \text{Leq} = 2(L+M) = 15 \text{ mH} \) to common-mode currents due to (the usually undesired) unintentional radiated signal.

• Thus differential-mode signal currents are passed more easily than the common-mode noise currents through this common-mode choke.
Simple Switching DC-DC Converter Analysis and Measurements
DC-DC Inverter with Common-Mode Choke on DC Power Cable

**DC Power Cable**
Carries Differential-Mode Currents & Common-Mode RF Noise Currents

Vin
6Vdc

Current Probe

Common-Mode Choke
Common-mode choke will be inserted and removed from circuit to observe its effect on common mode currents on the dc power cable as displayed on Spectrum Analyzer.

Spectrum Analyzer

D1
1N4004

R1
15 K

R2
15 K

Q1
TIP102

Q2
TIP102

Astable Push-Pull Blocking Oscillator
fosc = 11.6 kHz

Half-Wave Rectifier with Capacitor Filter

C1
0.01 uF

C2
0.01 uF

R215 K

C3
1000 uF

RLOAD
50 ohm
EG&G SCP-5(I)HF
(125 kHz – 500 MHz)
Snap On Current Probe
### SPECIFICATIONS

<table>
<thead>
<tr>
<th></th>
<th>SCP-1(a*) STD</th>
<th>SCP-5(a*) STD</th>
<th>SCP-5(a**) LF</th>
<th>SCP-5(a*) HF</th>
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<tr>
<td>Bandwidth, 3 dB</td>
<td>12 kHz to 140 MHz</td>
<td>50 kHz to 120 MHz</td>
<td>12 kHz to 70 MHz</td>
<td>125 kHz to 500 MHz</td>
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<tr>
<td>Points</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Bandpass Flatness, ±5 dB</td>
<td>36 kHz to 50 MHz</td>
<td>180 kHz to 100 MHz</td>
<td>35 kHz to 40 MHz</td>
<td>375 kHz to 150 MHz</td>
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<tr>
<td>Transfer Impedance, $Z_T$ (into 50Ω load)</td>
<td>1 V/A</td>
<td>5 V/A</td>
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<td>Electrostatic</td>
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<td>Shielding</td>
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<tr>
<td>at 10 MHz</td>
<td>-50 dB</td>
<td>-50 dB</td>
<td>.50 dB</td>
<td>-50 dB</td>
</tr>
<tr>
<td>at 100 MHz</td>
<td>-30 dB</td>
<td>-30 dB</td>
<td>-30 dB</td>
<td>-30 dB</td>
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<td>Typical Pulse</td>
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<tr>
<td>Input</td>
<td>100 amp - μs</td>
<td>20 amp - μs</td>
<td>20 amp - μs</td>
<td>20 amp - μs</td>
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<td>Probe Insertion Impédance</td>
<td>&lt;0.05Ω</td>
<td>&lt;1.0Ω</td>
<td>&lt;1.0Ω</td>
<td>&lt;2.0Ω</td>
</tr>
<tr>
<td>Probe Output</td>
<td>50Ω</td>
<td>50Ω</td>
<td>50Ω</td>
<td>50Ω</td>
</tr>
</tbody>
</table>

### Dimensions (cm)

![Diagram of dimensions](image)

- $a*$ = 1, 2, 3
- $a**$ = 1, 2, 3
- $A$ (cm) = 1.00, 2.00, 3.00
- $D$ (cm) = 6.63, 7.62, 8.64
- $T$ (cm) = 2.03, 2.03, 2.03
- $L$ (cm) = 0.70, 0.70, 0.70
- Mass (gm) = 184, 257, 273

### Typical Response

![Typical response graphs](image)
Astable Blocking Oscillator Analysis

When dc power is turned on, both BJTs turn on equally, as base-emitter current flows through R1 and R2. However, this is a potentially unstable circuit (like a ball resting on the crest of a hill). Imagine that there is a small positive noise glitch at the base of, say, Q2, that suddenly makes the base current of Q2 slightly greater than that of Q1. This will cause Q2 to conduct more than it was, lowering the voltage at the collector of Q2 and (since the voltage across C2 cannot change instantly), lowering the base voltage on Q1, causing Q1 to conduct less than it was. This raises the voltage at the collector of Q1, and this in turn makes Q2 conduct even harder. This positive feedback situation quickly drives Q2 in saturation and Q1 into cutoff.

Astable Push-Pull Blocking Oscillator
fosc = 11.6 kHz
Astable Blocking Oscillator Analysis

However Q2 does not remain saturated and Q1 does not remain cut off for long. This is because C2 charges through R2, and when C2 is charged high enough so that Vb1 exceeds Q1’s cut-in voltage, Vbe\text{CUTIN}, then Q1 turns on, and this causes Q2 to turn off. The process then reverses, with C1 charging until Vb2 exceeds Q2’s cut-in voltage, etc. Therefore continuous oscillation occurs, with Q1 and Q2 alternately changing between saturation and cut off. While Q2 is saturated during the first half of the oscillation period, current first flows from the center tap to the right terminal of the toroidal transformer, and then when Q1 saturates during the second half of the period, current flows from the center tap to the left terminal of the transformer, allowing a higher voltage to be induced in the secondary coil by transformer action.
To find the time it takes for $V_{b1}$ to increase from its initial value to its cut in value, we must think back to just before $Q2$ saturated. At this point, the voltage on the right side of $C2$ was $V_{in}$, and the voltage on the left side of $C2$ was $V_{be\_sat}$. Thus just before, and also just after, $Q2$ saturates,

$$V_{c2\_init} = V_{be\_sat} - V_{in}$$

since capacitor voltage cannot change instantly.
Astable Blocking Oscillator Analysis

Therefore, just after Q2 saturates, the value of Vbe1 is

\[ V_{\text{be1\_init}} = V_{c2\_init} + V_{ce\_sat} = (V_{be\_sat} - V_{\text{in}}) + V_{ce\_sat} \]

Vbe1 charges toward a value of \( V_{\text{be1\_final}} = V_{\text{in}} \), though it never gets there, since when \( V_{\text{be1}} = V_{\text{be\_cutin}} \), the state of the transistors change. Because \( R_2 = R_1 \) and \( C_2 = C_1 \), and the BJTs (Q1 and Q2) are matched, it takes \( 1/2 \) period = \( T/2 \) for Vbe1 to charge from \( V_{\text{be1\_init}} \) up to \( V_{\text{be\_cutin}} \).

Using the general first-order RC switching transient formula:

\[ V_x(t) = V_{x\_final} - (V_{x\_final} - V_{x\_initial})\exp(-t/(RC)) \]

we see that

\[ V_{\text{be1}}(t) = V_{\text{in}} - (V_{\text{in}} - (V_{be\_sat} - V_{\text{in}} + V_{ce\_sat})\exp(-t/R_2C_2) \]

When half a period elapses (\( t = T/2 \)), \( V_{\text{be1}}(t) = V_{\text{be\_cutin}} \), so

\[ V_{\text{be\_cutin}} = V_{\text{in}} - (V_{\text{in}} - (V_{be\_sat} - V_{\text{in}} + V_{ce\_sat})\exp\left(-\frac{T}{2\cdot R_2C_2}\right) \]
Astable Blocking Oscillator Analysis

\[ T = -2 \cdot \ln \left[ \frac{-(V_{be\_cutin} - V_{in})}{(2 \cdot V_{in} - V_{be\_sat} - V_{ce\_sat})} \right] \cdot R_2 \cdot C_2 \]

Since \( V_{in} \gg V_{be\_cutin}, V_{be\_sat}, V_{ce\_sat} \)

\[ T = 2 \cdot \ln(2) \cdot R_2 \cdot C_2 \]

\[ f_{osc} = \frac{1}{T} = \frac{1}{2 \cdot \ln(2) \cdot R_2 \cdot C_2} \]

Here \( R_2 := 15 \text{ k}\Omega \), \( C_2 := 10000 \text{ pF} \)

\[ f_{osc} = \frac{1}{2 \cdot \ln(2) \cdot R_2 \cdot C_2} \quad f_{osc} = 4.8089835 \times 10^3 \text{ Hz} \]

Note, the measured value of \( f_{osc} \) was 11 kHz, so the approximate result obtained from analysis is not accurate. This may be due to the fact that the simple analysis above did NOT take into account the effects of the inductive load.

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**Switching Frequency vs. DC-to-DC Conversion Efficiency**

- Conversion efficiency = $\frac{P_{\text{load}}}{P_{\text{in}}}$
- Conversion efficiency = $\frac{(V_{\text{in}} \cdot I_{\text{in}})}{(V_{\text{load}}(\text{avg})^2/R_{\text{load}})}$

<table>
<thead>
<tr>
<th>C1, C2 &amp; fosc</th>
<th>Pin = $V_{\text{in}} \cdot I_{\text{in}}$</th>
<th>Pout = $V_{\text{load}}^2/R_{\text{load}}$</th>
<th>Eff = $P_{\text{out}}/P_{\text{in}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.047 µF, 4.33 kHz</td>
<td>1.85 W</td>
<td>1.04 W</td>
<td>56.1%</td>
</tr>
<tr>
<td>0.01 µF, 11.6 kHz</td>
<td>2.14 W</td>
<td>1.31 W</td>
<td>61.1%</td>
</tr>
<tr>
<td>0.001 µF, 29.1 kHz</td>
<td>3.19 W</td>
<td>1.19 W</td>
<td>37.1%</td>
</tr>
</tbody>
</table>
Vb2(t) Measurement
Vc2 Measurement
Vsecondary Measurement (50 ohm load)
Vout Measurement (50 ohm load)
Without Common-Mode Choke
Current Probe Spectrum 0 – 20 MHz *without* Common-Mode Choke
With Common-Mode Choke Inserted
Current Probe Spectrum with Common-Mode Choke Inserted
Conducted Emissions Measurements

Basic Conducted Emissions Test Setup

- EQUIPMENT UNDER TEST
- Power Mains
- LINE IMPEDANCE STABILIZATION NETWORK (LISN)
- EMC ANALYZER
- TRANSIENT LIMITER
Line Isolation Stabilization Network (LISN)
LISN Spectrum (of either L1 or N lines) W/O common-mode choke
LISN Spectrum (Either L1 or N) with common-mode choke inserted
Experiment #2

Wireless FM Microphone
Goals

- Design and measure a 1.0 µH solenoidal air-core inductor
- Analyze and build an audio microphone amplifier circuit. (This and the next two items tie this EMC course back to the prerequisite electronics courses)
- Learn about the two conditions for oscillation in a feedback oscillator circuit.
- Learn how to analyze a typical RF “LC” oscillator circuit.
- Build/debug RF oscillator, then add audio modulation circuit to make a “wireless microphone”.
- Measure Harmonic Suppression.
- Experiment with radio wave propagation and different polarizations of radiated EM waves.
Equipment List

• DC power supply
• Agilent Spectrum Analyzer
• Agilent 0 – 100 MHz Digital Oscilloscope
• Agilent 0 – 20 MHz Function Generator
• Portable FM radio (Walkman style or boom box style)
• Tektronix Curve Tracer
Complete Circuit of FM Wireless Microphone

Electret Microphone
Bottom View

M1

Antenna (12" wire)

Rmic 10k

Cbypass1 0.001 UF

Rb1 470k

Cbypass2 0.001 UF

Rc1 560 ohms

Q1 2N3904

Q2 2N3904

Cmic 0.1 UF

Ccoup 0.1 UF

Vcc 9Vdc

Lx 1.0 uH

Rb2 10k

Ccoup 22 pF

Rmic 10k

Cfdbk 22 pF

Re2 1k

Cx 22 pF

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1 µH Inductor Design and Measurement

• Design a 1.0 µH air-core solenoidal-wound (single-layer) inductor.
• Use a suitable coil form (such as a felt-tip marker pen) and insulated hookup wire.
• Recall that this inductance formula is:

\[ L = \frac{N^2 \mu A}{l} \]

Where:
- \( N \) = Number of turns
- \( A \) = cross-sectional area of coil
- \( l \) = length of coil
- in air \( \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
Inductor Design

We desire an inductance of \( L := 1 \cdot \mu H \)

I chose a coil form with diameter \( d_{\text{form}} := 0.8 \text{ in} \)

I will adjust the coil length to be \( \text{len} := 1.2 \text{ in} \)

The permeability of free space (air) is \( \mu := 4 \cdot \pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \)

Find the cross-sectional area, \( A \)

\[
A := \pi \left( \frac{d_{\text{form}}}{2} \right)^2
\]

\[
A = 3.243 \times 10^{-4} \text{ m}^2
\]

\[
L = \frac{N^2 \cdot \mu \cdot A}{\text{len}}
\]

Solving for \( N \) we find \( N = 8.6 \).

Thus a coil with 9 turns and a length of 1.2 inches should yield an inductance of about 1 \( \mu H \).
Measuring Actual Value of Inductor

In this case

\[ C_{\text{known}} := 10 \times 10^3 \cdot \text{pF} \]

And a series resonant amplitude null was found at

\[ f_{\text{null}} := 1.52 \text{MHz} \]

\[ L_{\text{unknown}} := \frac{1}{4 \cdot C_{\text{known}} \cdot f_{\text{null}}^2 \cdot \pi^2} \]

\[ L_{\text{unknown}} = 1.096 \times 10^{-6} \text{ H} \]
Audio Amplifier Stage Analysis and Measurements

Bottom View

Q1 2N3904

M1 Electret Microphone

Cbypass1 0.001 UF

+9 V dc power bus

Vcc 9Vdc

Rmic 10k

Cmic 0.1UF

Rb1 470k

Rc1 560

Ccoup 0.1UF

Vaudio

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Beta Measurement using Curve Tracer
From Curve Tracer, $\beta = 160$
Calculation of DC Bias Q-Point of Audio Stage

To find the dc bias "Q" point, first find the quiescent base current:

\[ Ibq := \frac{9 \cdot V - 0.7 \cdot V}{470 \cdot k\Omega} \]

\[ Ibq = 1.766 \times 10^{-5} \text{ A} \]

Then assuming Q1 is forward active, the collector current is

\[ \beta := 160 \quad \text{Icq := } \beta \cdot \text{Ibq} \]

\[ \text{Icq} = 2.826 \times 10^{-3} \text{ A} \]

Therefore the Q-point of the audio stage is

\[ \text{Icq} = 2.826 \times 10^{-3} \text{ A} \]

and

\[ \text{Vceq := } 9 \cdot V - \text{Icq} \cdot 560 \cdot \Omega \]

\[ \text{Vceq} = 7.418 \text{ V} \]
Measured Q-Point of Audio Stage

- Measured $V_{ce1q} = 7.46\ V$ (Predicted 7.42V)
- Measured $I_{c1q} = 2.75\ mA$ (Predicted 2.83 mA)
Calculated AC Gain of Audio Stage

AC Model of Audio Stage

\[ vo(t) = -(vi(t)/rpi) \times \beta \times Rc \]

\[ Av = vo(t)/vi(t0) = -\beta \times Rc/rpi \]
Calculated and Measured AC Gain of Audio Stage

\[ r_{pi} := \frac{26 \text{ mV}}{I_{bq}} \]

\[ r_{pi} = 1.472 \times 10^3 \, \Omega \]

\[ A_v = \frac{v_o(t)}{v_i(t)} = -\beta \cdot R_c \]

\[ A_v := \frac{-160.560}{1.472 \times 10^3} \]

\[ A_v = -60.87 \]

Measured AC gain (Using function generator in place of microphone, set to 5 kHz and 10 mV amplitude) is

\[ A_{v_{\text{observed}}} = -52 \quad (\text{Calculated } A_v = -60.9) \]

*Note: we used a highly simplified small-signal BJT model.*
Analysis of RF Oscillator Circuit

- **Vcc**: +9 V dc power bus
- **C bypass1**: 0.001 UF
- **Rb2**: 10k
- **Lx**: 1.0 uH
- **Q2**: 2N3904
- **C bypass2**: 0.001 UF
- **C fb**: 22 pF
- **Re2**: 1k
- **Antenna**: (12" wire)
DC Q-Point Calculation of Q2

- The β of Q2 was measured on the curve tracer and found to be β = 160.

\[
I_{b2q} := \frac{9.0 \, V - 0.7 \, V}{10 \cdot k\Omega + (160 + 1) \cdot 1 \cdot k\Omega} \quad I_{b2q} = 4.854 \times 10^{-5} \, A
\]

\[
I_{c2q} := 160 \cdot I_{b2q} \quad I_{c2q} = 7.766 \times 10^{-3} \, A
\]

\[
V_{ce2q} := 9 \cdot V - (160 + 1) \cdot 1 \cdot k\Omega \cdot I_{b2q} \quad V_{ce2q} = 1.185V
\]

When \( V_{ce2q} \) was measured (first Cfdbk was removed so that the circuit was not oscillating.) it was found that \( V_{ce2q} = 1.27 \, V \) (quite close to predicted value of 1.185 \( V \)), and \( I_{c2q} = 7.73 \, mA \) (quite close to the predicted value of 7.77 \( mA \)).
AC Model of RF Oscillator

• In making this model, we assume that Cbypass1 and Cbypass2 (both 0.001 µF) act like short circuits at the 34 MHz oscillation frequency since the magnitude of the impedance of these capacitors at 34 MHz is $1/(2\pi*34 \text{ MHz}*0.001 \text{ µF}) = 4.82 \Omega$!

• But note that at audio frequencies, Cbypass1 and Cbypass2 act like open circuits, because the magnitude of the impedance at 1 kHz is $1/(2\pi*1 \text{ kHz}*0.001 \text{ µF}) = 159.2 \text{ kΩ}$!

• This is important so that the audio modulating signal applied to the base of Q2 from the audio amplifier stage is not shorted out by Cbypass2.

• In the AC model of Q2, $\beta = 160$

• In the AC model of Q2, $r_{pi2} = 26 \text{ mV} / I_{b2q} = 535.6 \text{ ohms}$
AC Model 34 MHz Oscillator Circuit

The base is grounded because of C bypass2 acts like a short circuit at the 34 MHz oscillation frequency.
Oscillator Analysis

- Q2 functions as a “common base” amplifier.
- The input signal voltage is delivered to the emitter terminal (E), creates a base current \( i_{b2}(t) = -\frac{v_E(t)}{r_{pi2}} \), and the amplified output appears at the collector terminal (C).
- Note that the output is fed back to the input via a frequency-selective feedback network that consists of \( R_{e2}, C_{fdbk}, L_x, \) and \( C_x \).
Frequency of Oscillation set by $R_x, C_x$ parallel resonant (tank) circuit

$$Z_{RxCx} = \frac{1}{j2\pi fCx + \frac{1}{j2\pi fLx}}$$

This impedance becomes infinite (acts like an open circuit) when its denominator is set to zero

$$j2\pi fCx + \frac{1}{j2\pi fLx} = 0$$

$$f_{res} = \frac{1}{2\pi \sqrt{LxCx}} = \frac{1}{2\pi \sqrt{10^{-6} \cdot 22 \times 10^{-12}}} = 33.93 \text{ MHz}$$
A general sinusoidal feedback oscillator consists of an amplifier and some sort of an external feedback network.

\[ \text{Amplifier Voltage Gain } A(f) \]
\[ \text{Feedback Network Voltage Gain } \beta(f) \]

\[ \Rightarrow \text{ Loop Gain } = A(f) \beta(f) \]
Conditions for Oscillation:

(a) The magnitude of the voltage gain around the loop (loop gain) is greater than 1, so that noise at frequency “f” that is present due to the power-up transient will be amplified to higher and higher levels as it circulated around the frequency-selective feedback loop. (The oscillations do not build up forever, since eventually the BJT is driven into saturation or cutoff. The oscillation amplitude is self-limiting due to device nonlinearities.)

\[ |A(f)\beta(f)| > 1.0 \]

(b) The phase shift around the loop must be an integral multiple of \(2\pi\) radians, so that the fed back sinusoidal signal will add “in phase” (constructively) with the signal already at the input, and then oscillations can then build up.

Phase Angle \([A(f)\beta(f)] = n2\pi\), where \(n\) is any integer
Breaking the Feedback Loop: Loop Gain

\[ \text{Loop Gain} = \beta(f) A(f) = \frac{V_{out}}{V_{in}} \]

Amplifier Voltage Gain \( A(f) \)

Feedback Network Voltage Gain \( \beta(f) \)

\( V_{out} \) and \( V_{in} \) are connected through a closed loop, with feedback network voltage gain \( \beta(f) \). Breaking the feedback loop results in the loop gain being \( \beta(f) A(f) \).
Loop Gain Calculation using AC Model of RF Oscillator

Add Rx in series with Lx to model a “real” inductor more accurately.
We may write node equations at the emitter node \( (V_E) \) and the collector node \( (V_{out}) \) in terms of the Laplace complex frequency variable \( \text{“s”} \).

\[
eq 1 := \frac{V_{out}}{s L_x + R_x} + V_{out} s C_x - \frac{\beta_2 V_E}{\text{rpi} 2} = 0
\]

\[
eq 2 := \frac{V_E}{\text{rpi} 2} + \frac{V_E}{\text{re} 2} + (V_E - \text{Vin}) s C_{\text{fdba}} + \frac{\beta_2 V_E}{\text{rpi} 2} = 0
\]

Then eliminating \( V_E \), we can show that

\[
\text{LoopGain} = \frac{V_{out(s)}}{V_{in(s)}} = \frac{s C_{fdba} R_{e2} \beta_2 (s L_x + R_x)}{s^2 C_x L_x + s C_x R_x + 1)(s C_{fdba} R_{e2} r_\pi + [r_\pi + (\beta + 1)R_{e2}])}
\]

Replace \( s \) by \( j 2\pi f \) and plot the magnitude and angle of the loop gain of this circuit in order to find the frequency(s) at which this circuit can oscillate.
|Loop Gain|

Angle(Loop Gain)
(degrees)

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Loop Gain phase shift passes through 0 degrees at $f = 34.35146$ MHz, where the Loop Gain amplitude is $40.30 > 1$ => circuit oscillates
Now connect the audio stage to the RF oscillator stage by adding Ccoup

Final Circuit

Electret Microphone

Bottom View

Antenna (12" wire)

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Final Construction on Breadboard

*Note* wires must be *KEPT VERY SHORT*
**FM Modulation Method**

- The audio waveform is capacitively coupled onto the base of RF oscillator Q2.
- This causes a small degree of base-width modulation at the audio rate.
- This causes the collector-to-ground capacitance of Q2 exhibited by the BJT to slightly vary at this audio rate.
- Because $C_x$ is in parallel with Q2’s collector-to-ground capacitance, this causes the resonant frequency of the tank circuit ($L_x, C_x$) to be varied slightly, and thus the oscillation frequency is varied back and forth very slightly about the nominal oscillation frequency at the audio rate.
Oscilloscope Waveform

(Observed by VERY LIGHTLY capacitively coupling the scope probe by clipping the probe onto the plastic wire of Lx near the collector of Q2 – if the probe is clipped directly on the collector, it may load down the oscillator and stop oscillation)
Oscilloscope Waveform Observations

- From the oscilloscope, we see that the frequency of oscillation is near 34 MHz.
- The coil may be pulled apart or pushed back together to alter the resonant frequency as desired.
- Note that the waveform is quite distorted, therefore, we expect large harmonics to be present.
Spectrum Analyzer Waveform
(observed using a 12” wire antenna connected to the input of the Spectrum Analyzer and placed near the wireless FM microphone.)
Spectrum Analyzer Observations

- Because 12” antenna wires were used on both the spectrum analyzer and the wireless FM microphone, the received 2nd harmonic (at 67.5 MHz) was 10 dB above the fundamental (at 33.7 MHz).
- The 3rd harmonic (at 100.5 MHz) was 3 dB below the 2nd harmonic.
- The 4th harmonic was about equal in strength to the 3rd harmonic.
- The higher harmonics rapidly fell off in amplitude.
- We will receive the 3rd harmonic on the FM receiver.
Hearing the 3\textsuperscript{rd} harmonic on an FM radio

- Circuit can pick up speech clearly from 10 or more feet away from the microphone.
- Frequency of oscillation is not very stable, since it is determined by $L_x,C_x$. Moving hand near coil will detune.
- Range of wireless microphone circuit is only about 40 feet.
- Can experiment with different antenna orientations and antenna lengths.
- Quarter-wave monopole would be about $(300/100)/4 = \frac{3}{4}$ meter in length --- this should be a good antenna length for the 3\textsuperscript{rd} harmonic signal.
Is it possible to make a 102 MHz oscillator on a breadboard possible with a “lowly” mundane 2N3904 BJT? **YES!**

- Replace Lx with a SINGLE LOOP of wire (perhaps 1.5” in length).
- Note using the scope and/or the spectrum analyzer that the circuit still oscillates.
- Adjust the loop length for a signal in the FM band (88 MHz – 108 MHz)
102 MHz FM Wireless Microphone

NOTE: *What a difference a SHORT wire can make in an RF circuit!*
Oscilloscope Waveform

(Note the freq of oscillation is now about 102 MHz. Because this is a 100 MHz scope, the sine wave looks “clean”…no harmonics appear to be present!)
Spectrum of 102 MHz Wireless Microphone
Conclusion

The 102 MHz wireless microphone signal should now transmit further than the 34 MHz one, since we are now listening to the (stronger) fundamental frequency, rather than having to listen to the 3rd harmonic of a 34 MHz fundamental frequency!

Some team’s breadboards may not permit oscillation directly at 102 MHz, although other team’s breadboards (usually the ones built as neatly as possible, with the shortest leads!) should still function at this higher frequency!
Experiment #3

Bigger is not always better!: Benefits of DC Power Bus Capacitor Bypassing on Vcc(t) and on Radiated Emissions
Equipment List

- Agilent 54624A 100 MHz Digital Oscilloscope
- Agilent E4402B Spectrum Analyzer (100 Hz – 3 GHz)
- Agilent E3631A Variable Triple Output DC Power Supply
- Prototyping Breadboard
- Assorted capacitors with short leads (10 μF, 0.1 μF, 0.001 μF (1 nF), 100 pF)
- 74HC04 High Speed CMOS Hex Inverter
Goals

• Familiarize student with the ring oscillator, and propagation time, rise time, and fall time measurements.
• Allow student to investigate effects of various types of bypass capacitors on radiated EMC emissions.
• Allow student to investigate effects of various types of bypass capacitors on Vcc(t) dc power bus waveform.
• Learn how to use the spectrum analyzer/tracking generator to model a real capacitor in terms of an ideal capacitor in series with an ideal inductor
Part 1: Ring Oscillator

- Construct “ring oscillator” on breadboard
- Keep all leads as short and as direct as possible, as shown in the following breadboard layout.
- Connect the +5 V dc power supply lines (Vcc and GND) to the 74HC04 hex inverter integrated circuit using two power distribution rails on your breadboard, Vcc = +5V at the top, and GND = 0V at the bottom, as shown in the following photograph.
- Note from this photograph how the dc power bus “ac bypass” capacitor (C_bypass = 1 nF) is connected as close as physically possible to the Vcc and GND pins of the 74HC04.
- Note that the capacitor should have its leads cut as short as possible to minimize lead inductance.
3-Inverter Ring Oscillator Circuit

Vcc = +5 V

U1A

U1B

U1C

U1D

74HC04

74HC04

74HC04

74HC04
Theory of operation – N-inverter Ring Oscillator (N odd and here N=3)

• After a change at the output, it takes $N \times T_{prop}$ seconds for this change to propagate back to the output, causing the output to change state.
• Thus $T_{osc} = 2(N \times T_{prop})$
• $F_{osc} = 1/T_{osc} = 1/(2N \times T_{prop})$
• $T_{prop} = 1/(2N \times F_{osc})$
• Rightmost inverter is not part of ring. It is used to “buffer” the output (reduce capacitive loading that might slow down the final stage in the ring.)
Ring Oscillator Breadboard Layout

Note: All wires kept SHORT!
Ring Oscillator Output Waveform
Calculate Tprop of each 74HC04 inverter using the measured fosc displayed on the oscilloscope

• $2 \times (3 \times T_{\text{prop}}) = 1/35.34$ MHz
• $T_{\text{prop}} = 4.72$ ns
Now connect five inverters in a ring, using the sixth inverter as an isolating buffer to reduce capacitive loading on one of the oscillating inverters in the ring, and thereby alter the speed of oscillation.

Use the measured value of $T_{\text{prop}}$ from the 3-inverter oscillator to predict the value of $f_{\text{osc}}$ of the 5-inverter ring oscillator.
5-inverter Ring Oscillator Waveform
Calculation of fosc from measured value of Tprop

Predicting oscillation frequency from Tprop measured earlier

\[ Fosc = \frac{1}{2 \times 5 \times Tpd} = \frac{1}{10 \times 4.72 \text{ ns}} = 21.2 \text{ MHz} \]

This is quite close to the observed value of fosc = 21.8 MHz
Part 2. Effect of dc power bus bypass capacitors on radiated emissions
3-inverter ring oscillator
0-300 MHz Spectrum (12” wire antenna on spectrum analyzer) Cbypass = 1 nF
3-inverter ring oscillator
0-300 MHz Spectrum (12” wire antenna on spectrum analyzer) Cbypass = 100 pF
3-inverter ring oscillator
0-300 MHz Spectrum (12” wire antenna on spectrum analyzer) Cbypass = 4.7 µF
Conclusions

• Higher value dc power bus bypass capacitors appear to limit EMC emissions.
• But this is because these higher capacitors permit more Vcc(t) dc power bus noise glitches, which significantly slow the rise and fall times of the output signal from the ring oscillator.
Part 3. Effect of dc power bus bypass capacitors on $V_{cc}(t)$ dc power bus spikes
3-inverter ring oscillator
Oscilloscope between Vcc and ground (near power terminal) Cbypass = 100 pF. Vnoise = 20 mV pp
3-inverter ring oscillator
Oscilloscope between Vcc and ground (near power terminal) Cbypass = 1 nF  Vnoise = 29 mV pp
3-inverter ring oscillator
Oscilloscope between Vcc and ground (near power terminal) Cbypass = 100 nF  Vnoise = 26 mV
3-inverter ring oscillator
Oscilloscope between Vcc and ground (near power terminal) C\_bypass = 4.7 \mu F V\_noise = 28 mV
Conclusions

• 100 pF capacitor does the best job of reducing the high-frequency Vcc(t) glitches for this 35 MHz ring oscillator.

• This results in squarer output switching waveforms, and thus results in higher undesired radiation.

• In a general digital system that encompasses signals of many different frequencies, probably a parallel combination of several dc power bus bypass capacitors would be best for reducing Vcc(t) glitches of both high and low frequency.
Part 4.

Using Spectrum Analyzer/Tracking Generator to measure the self-resonant frequency, and also $L_x, C_x$ of a “real” capacitor.
Tracking Generator

- Built into our spectrum analyzer.
- The spectrum analyzer’s continuously varying local oscillator (LO) signal is mixed with the analyzer’s IF frequency.
- This produces an output frequency (available to the user) that matches, or “tracks”, the frequency to which the spectrum analyzer is currently tuned.
- Therefore if the tracking generator output (TGout) is connected directly to the spectrum analyzer’s input (RFin), a flat horizontal line will be traced.
- If a 2-port circuit or device under test (DUT) is placed between TGout and RFin, a “stimulus – response”, or “frequency response” curve will be traced, allowing us to automatically measure how well the DUT passes signals at various frequencies.
Features
Stimulus Response: Tracking Generator

Source

DUT

3.6 GHz BPF
f_{LO}=4.6 GHz

Spectrum Analyzer

3.6 GHz

IF

LO

CRT Display

Tracking Generator

DUT

RF in

TG out

DUT

Display

fin=1GHz

Fout=4.6-3.6=1GHz
Capacitance Measurement using Spectrum Analyzer/Tracking Gen

Low Freq Model (well below self-resonance)

High Freq Model (in the vicinity of self-resonance)

“Real Data” recorded using our lab spectrum analyzer using a 20% tolerance capacitor marked “103” = 10000 pF = 10 nF.

18 dB Atten at 4 MHz

Self Resonance F=13.2 MHz

Linear Frequency Scale used!
Measuring Capacitor’s C₁ & L₁

Using the "Well below resonance model"

At dc (ω = 0) the input voltage source suffers the following attenuation as it arrives at the output terminals of the tracking generator (across the capacitor):

\[ \text{Attend}B_0 = 20 \cdot \log \left( \frac{V_{\text{gen}}}{V_c} \right) \]

\[ \text{Attend}B_0 := 20 \cdot \log \left( \frac{50 \cdot \Omega + 50 \cdot \Omega}{50 \cdot \Omega} \right) \]

\[ \text{Attend}B_0 = 6.021 \text{ dB} \]

As ω increases from 0 to frequency "ωₓ", the output falls by an additional "ArelativedB" decibels, which may be conveniently measured using the spectrum analyzer. Thus the overall attenuation "AttendBₓ" at frequency ω = ωₓ is given by

\[ \text{Attend}B_x = \text{ArelativedB} + \text{Attend}B_0 = 20 \cdot \log \left( \frac{V_{\text{gen}}}{V_c} \right) \]

\[ \text{Gain} = \frac{V_{\text{out}}}{V_{\text{in}}} \]

\[ \text{Attenuation} = \frac{1}{\text{Gain}} = \frac{V_{\text{in}}}{V_{\text{out}}} \]

\[ \text{ArelativedB} + 6.021 = 20 \cdot \log \left( \frac{1}{\frac{1}{50} + j \cdot \omega_x \cdot C_1} \right) \]

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\[
A_{\text{relativeB}} + 6.021 = 20 \log \left( |2 + 50 \cdot j \cdot \omega_x \cdot C_1| \right)
\]

\[
\frac{(A_{\text{relativeB}}+6.021)}{10^{20}} = \sqrt{2^2 + (50 \cdot \omega_x \cdot C_1)^2}
\]

\[
10 \left( \frac{A_{\text{relativeB}}+6.021}{10} \right) = 2^2 + (50 \cdot \omega_x \cdot C_1)^2
\]

\[
C_1 = \frac{1}{50 \cdot \omega_x} \sqrt{10^{20} - 4}
\]

Important Result that we will use in the lab!
Then we may use the "near resonance" model to find $L_1$ in terms of the resonant frequency where the output voltage amplitude passes through a resonance "dip" that corresponds to the frequency $\omega = \omega_{\text{res}}$ at which the impedance of "$C_1$" and the impedance of the parasitic lead inductance "$L_1$" cancel. This frequency may be expressed in terms of $L_1$ and $C_1$ as shown below:

$$\frac{1}{j \cdot \omega_{\text{res}} \cdot C_1} + j \cdot \omega_{\text{res}} \cdot L_1 = 0$$

$$\omega_{\text{res}} = \frac{1}{\sqrt{L_1 \cdot C_1}}$$

$$f_{\text{res}} = \frac{1}{2 \cdot \pi \cdot \sqrt{L_1 \cdot C_1}}$$
In our example the capacitor was marked "10 nF", and from the spectrum analyzer display, at a frequency well below resonance (4 MHz), we measured

\[
A_{\text{relative dB}} := 18.0 \text{ dB at frequency } \omega_x := 2 \cdot \pi \cdot 4 \cdot 10^6 \frac{\text{r}}{\text{s}}
\]

\[
C_1 := \frac{1}{50 \cdot \omega_x} \cdot \sqrt{10^{\frac{A_{\text{relative dB}} - 6.021}{10}} - 4}
\]

\[
C_1 = 1.254 \times 10^{-8} \text{ F (or 12.54 nF)}
\]

The self resonant frequency was observed to be 13.2 MHz, so

\[
13.2 \cdot 10^6 = \frac{1}{2 \cdot \pi \cdot \sqrt{L_1 \cdot C_1}}
\]

Solving for \(L_1\), we find

\[
L_1 := 11.6 \text{ nH}
\]
## Typical Bypass Capacitor Self-Resonant Frequency and also C1,L1

<table>
<thead>
<tr>
<th>Marked Value</th>
<th>Lead Length cm</th>
<th>Arelative dB</th>
<th>fx MHz</th>
<th>fres MHz</th>
<th>C1</th>
<th>L1 nH</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 pF</td>
<td>0.5</td>
<td>1.54</td>
<td>40.5</td>
<td>80</td>
<td>102 pF</td>
<td>10.6</td>
</tr>
<tr>
<td>1 nF</td>
<td>0.5</td>
<td>1.73</td>
<td>4.58</td>
<td>27.9</td>
<td>0.97 nF</td>
<td>33.5</td>
</tr>
<tr>
<td>1 nF</td>
<td>2</td>
<td>1.79</td>
<td>4.58</td>
<td>22.9</td>
<td>0.99 nF</td>
<td>48.7</td>
</tr>
<tr>
<td>0.1 uF</td>
<td>0.5</td>
<td>5.6</td>
<td>0.103</td>
<td>4.27</td>
<td>0.100 uF</td>
<td>13.9</td>
</tr>
<tr>
<td>0.1 uF</td>
<td>2</td>
<td>7.26</td>
<td>124</td>
<td>3.55</td>
<td>0.107 uF</td>
<td>18.8</td>
</tr>
<tr>
<td>0.33 uF</td>
<td>0.5</td>
<td>17.8</td>
<td>0.200</td>
<td>1.45</td>
<td>0.245 uF</td>
<td>49.1</td>
</tr>
<tr>
<td>0.33 uF</td>
<td>2</td>
<td>12.1</td>
<td>0.125</td>
<td>1.15</td>
<td>0.200 uF</td>
<td>96.4</td>
</tr>
</tbody>
</table>
Why was the 100 pF bypass capacitor more effective than the 0.1 uF bypass capacitor?

- The 35 MHz ring oscillator used in this lab causes narrow Vcc(t) power supply glitches that repeat at a 35 MHz rate.

- A narrow 35 MHz pulse train has significant spectral components concentrated at harmonic frequencies of n*35 MHz, where n = 1, 2,3,4,5,…

- A real bypass capacitor can only bypass noise harmonics that are well below its self-resonant frequency. This is because it must exhibit relatively low reactance (compared to the load impedance being driven) at the noise harmonic frequency of interest.
• For noise harmonics below a real capacitor’s self-resonant frequency, the capacitor exhibits a negative (capacitive) impedance, and thus it behaves like a capacitor.

• For noise harmonics above its self-resonant frequency, the real capacitor exhibits positive reactance, and thus behaves like an inductor --- which means the real capacitor does NOT bypass this noise effectively
• Note that an ideal 100 pF capacitor exhibits reasonably low reactance at even the lowest (fundamental) noise frequency: $X_c(35 \text{ MHz}) = 1/(2\pi \times 35 \text{ MHz} \times 100 \text{ pF}) = 45 \text{ ohms}$, and for the nth harmonic, $X_c = 45/n \text{ ohms}$.

• Also, the “real” version of this capacitor has a relatively high self-resonant frequency (80 MHz), so it can be expected to do a good job filtering out at least the fundamental frequency component (35 MHz) and the second harmonic component (70 MHz).
• An ideal 0.1 uF capacitor has a much lower impedance, $X_c$ at 35 MHz, but the self-resonant frequency of the real version of this capacitor is only about 4 MHz, which is much lower than even the fundamental noise frequency of 35 MHz!

• Thus even though an IDEAL version of a 0.1 uF capacitor would do a better job than the 100 pF capacitor in removing power supply noise, because of its relatively low self-resonant frequency, the real 0.1 uF capacitor is incapable of filtering even the 35 MHz fundamental component of the Vcc(t) noise!
When it comes to choosing a dc power bus Bypass Capacitor…

• **Conclusion:**

  Bigger is *not* always better!